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Abstract. This study deals with a logic TIL of temporal information. By formalizing a language of logic TIL containing classical propositional operations, modal operations determined by parameters and tense operations, we provide a logical tool for solving definability problem of temporal information. Finally, we show that logic TIL is decidable.

1. Introduction

Logical treatment of knowledge representation has been an issue of concern [2, 4, 8]. But in many approaches, models for the formalized languages have been regarded as ones of the current states of knowledge. In real world, however, temporal information, for example hospital records, meteorological phenomena and so on, universally exist, and hence files in their databases are dynamic and must be brough up to date regularly to reflect changes in status of items in the files.

On the other hand, since information processing presupposes the classification of information, a class of relations in a set of objects, which are determined by properties of the objects may be looked upon as criterions of the classification. The main task of temporal information processing is to collect all relevant information characterized by some of the given properties in a domain at a moment of time.

The purpose of this paper is to solve the definability problem of temoral information based on a class of relations — indiscernibility relations in the set of objects, by proposing a language, and the decision problem for logic TIL. Two objects are in such an indiscernibility relation corresponding to a set of properties exactly if the two are distinguishable by means of the set of properties. The indiscernibility (cf.[10]) is generalization of some properties of real-world models, say, relation databases [14], information systems [12]. The logic considered in this paper is based on tense logic [1, 7, 13] and modal logic [5, 6]. The former will be used as an auxiliary tool of this approach; the latter has modal operators relative to the parameters referred to as properties of objects.

2. Temporal information system and temporal definability of objects

For convenience, we introduce a mathematical frame for temporal information — a temporal information system. By a *temporal information system* (a TI system for short) we mean a system $\mathbf{S}$=(OB, PROP, TM, $\{ind(P,t)\}_{P\in PROP, t\in TM}$, R) where OB is a nonempty set of objects, PROP is the family of all sets of properties of objects, TM is a nonempty set of moments of time, ind(P,t) is an equivalence relation in OB, referred to as *temporal indiscernibility relation* in OB determined by properties from P at time t ($\in$TM), R is a transitive linear ordering in TM.

Given a TI system, we define *equivalence class* ind(P, t)(o) of an object o determined by relation ind(P, t):

$$ind(P,t)(o)=\{o'\in OB \mid (o,o')\in ind(P,t)\}.$$

Theorem 2.1. The following conditions are equivalent:

(1) $ind(P,t)\subseteq ind(Q,t)$.

(2) for all $o\in OB$ $ind(P,t)(o)\subseteq ind(Q,t)(o)$.

For $P\in PROP$ and $S\subseteq OB$, we define $\underline{ind}(P,t)S=\{o\in OB \mid ind(P,t)(o)\subseteq S\}$ and $\overline{ind}(P,t)S=\{o\in OB \mid ind(P,t)(o)\cap S\neq\varnothing\}$. In other words, $\underline{ind}(P,t)S$ is the union of those equivalence classes of ind(P,t) which are included in S; $\overline{ind}(P,t)S$ is the union of those equivalence classes of ind(P,t) which have an element in common with S. $\underline{ind}(P,t)S$ is called a *lower approximation* of S with respect to relation ind(P,t), and $\overline{ind}(P,t)S$ an *upper approximation* of S with respect to ind(P,t).

Temporal indiscernibility of objects influences definability of sets of objects by means of properties.

We say that set S is (P,t)-*definable* exactly if $\underline{ind}(P,t)S=S=\overline{ind}(P,t)S$, i.e., S is (P,t) -definable iff it is covered by equivalence classes of relation ind(P,t). Given a set S of objects, in terms of approximation we define (P,t)-*positive*, (P,t)-*negative* and (P,t)-*borderline* elements of S as follows:

— o is a (P,t)-positive element of S iff $o\in \underline{ind}(P,t)S$,

— o is a (P,t)-negative element of S iff $o\in OB-\overline{ind}(P,t)S$,

— o is a (P,t)-borderline element of S iff $o\in \overline{ind}(P,t)S-\underline{ind}(P,t)S$.

Theorem 2.2.

(1) $\underline{ind}(P,t)S\subseteq S\subseteq\overline{ind}(P,t)S$.

(2) $\underline{ind}(P,t)S=-\overline{ind}(P,t)-S$.

(3) $\overline{ind}(P,t)S=-\underline{ind}(P,t)-S$.

Theorem 2.3.

(1) If $ind(P,t)\subseteq ind(Q,t)$, then for any $S\subseteq OB$, $\underline{ind}(Q,t)S\subseteq\underline{ind}(P,t)S$ and

$\overline{ind}(P,t)S \subseteq \overline{ind}(Q,t)S$.

(2) If $ind(P,t) \subseteq ind(Q,t)$, then (Q,t)-definability implies (P,t)-definability.

Proof. (1) This proof is the application of Theorem 2.1.

(2) This proof is by Theorem 2.2(1) and condition (1). //

We list some of the properties of temporal approximations:

Theorem 2.4.

(1) $S \subseteq \underline{ind}(P,t)\overline{ind}(P,t)S$.

(2) $\underline{ind}(P,t)(S \cap T) = \underline{ind}(P,t)S \cap \underline{ind}(P,t)T$.

(3) $\underline{ind}(P,t)S \cup \underline{ind}(P,t)T \subseteq \underline{ind}(P,t)(S \cup T)$.

(4) $\underline{ind}(P,t)OB = OB$.

(5) $\underline{ind}(P,t)\varnothing = \varnothing$.

(6) if $S \subseteq T$, then $\underline{ind}(P,t)S \subseteq \underline{ind}(P,t)T$.

(7) $\underline{ind}(P,t)S = \underline{ind}(P,t)\underline{ind}(P,t)S$.

Note that the counterparts of the above conditions for upper approximations hold by duality of lower and upper approximations.

3. The logic TIL of temporal information

Formulas of the language of TIL are intended to represent sets of objects. We admit the following pairwise disjoint sets to define the formulas:

— VARO: a set of variables representing sets of objects,

— $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$: the set of propositional operators of negation, disjunction, conjunction, implication and equivalence, respectively,

— VARP: a set of variables representing sets of properties,

— $\{[\,], <\,>\}$: the set of modal propositional operators corresponding to lower and upper approximations, respectively,

— $\{G, F, H, P\}$: the set of tense operators,

— $\{(,)\}$: the set of left and right parentheses as punctuations.

Set FOR of the formulas is the least set satisfying the following conditions:

$VARO \subseteq FOR$

if $X, Y \in FOR$, then $\neg X$, $X \vee Y$, $X \wedge Y$, $X \rightarrow Y$, $X \leftrightarrow Y$, GX, FX, HX, $PX \in FOR$,

if $A \in VARP$ and $X \in FOR$, then $[A]X$, $<A>X \in FOR$.

Formulas X and Y are called *subformulas* of foumulas $X \vee Y$, $X \wedge Y$, $X \rightarrow Y$ and $X \leftrightarrow Y$; X is called subformula of formulas $\neg X$, GX, FX, HX, PX, $[A]X$ and $<A>X$; in addition, any formula X is a subformula of itself.

Before we formally give the semantic definitions of G, F, H and P, let us intuitively interpret them as: necessarily in the future, possibly in the future , necessarily in the past and possibly in the past , respectively.

Truth or falsity of a formula, to a certain extent, depends on time. From this viewpoint, we define semantics of the language by means of the notions of *model* determined by a TI system and *temporal satisfiability* of formulas in a model.

By a model, we mean a system $M=(OB, TM, PROP, \{ind(P,t)\}_{P\in PROP, t\in TM}, R, v)$ where OB is a nonempty set of objects, TM is a nonempty set of moments of time, PROP is the family of all sets of properties of objects, ind(P,t) is the indiscernibility relation in OB, determined by properties from P at time t, R is a transitive linear ordering in TM, v is a valuation function such that $v(A)\in PROP$ for $A\in VARP$, $v(X)\subseteq OB$ for $X\in VARO$.

Given a model M, we define temporal satisfiability of the formulas. For $o\in OB$ and $t\in TM$, we say that an object o satisfies a formula X in model M at moment of time t ($M \models_o^t X$) iff the following conditions are satisfied:

for $X\in VARO$ $M\models_o^t X$ iff $o\in v(X)$,
$M\models_o^t X\vee Y$ iff $M\models_o^t X$ or $M\models_o^t Y$,
$M\models_o^t X\wedge Y$ iff $M\models_o^t X$ and $M\models_o^t Y$,
$M\models_o^t \neg X$ iff not $M\models_o^t X$,
$M\models_o^t X\rightarrow Y$ iff $M\models_o^t \neg X\vee Y$,
$M\models_o^t X\leftrightarrow Y$ iff $M\models_o^t (X\rightarrow Y)\wedge(Y\rightarrow X)$,
$M\models_o^t [A]X$ iff for all o' $(o,o')\in ind(v(A),t)$ implies $M\models_{o'}^t X$,
$M\models_o^t \langle A\rangle X$ iff there is an o' such that $(o,o')\in ind(v(A),t)$ and $M\models_{o'}^t X$,
$M\models_o^t GX$ iff for all t' $(t,t')\in R$ implies $M\models_o^{t'} X$,
$M\models_o^t FX$ iff there is a t' such that $(t,t')\in R$ and $M\models_o^{t'} X$,
$M\models_o^t HX$ iff for all t' $(t',t)\in R$ implies $M\models_o^{t'} X$,
$M\models_o^t PX$ iff there is a t' such that $(t',t)\in R$ and $M\models_o^{t'} X$.

It is easily seen that operators [A], G and H are dual with respect to <A>, F, and P, respectively. To each formula X we assign a set $ext_M^t X$ (*extension* of X in model M at time t) of these objects which satisfy X in M at time t: $ext_M^t X=\{o\in OB \mid M\models_o^t X\}$.

Theorem 3.1.
(1) $ext_M^t X=v(X)$ for $X\in VARO$.
(2) $ext_M^t \neg X=-ext_M^t X$.
(3) $ext_M^t X\vee Y=ext_M^t X\cup ext_M^t Y$.

(4) $ext^t_M X \wedge Y = ext^t_M X \cap ext^t_M Y$.
(5) $ext^t_M X \rightarrow Y = -ext^t_M X \cup ext^t_M Y$.
(6) $ext^t_M X \leftrightarrow Y = (ext^t_M X \cap ext^t_M Y) \cup ((-ext^t_M X) \cap (-ext^t_M Y))$.
(7) $ext^t_M [A]X = \underline{ind}(v(A),t) ext^t_M X$.
(8) $ext^t_M <A>X = \overline{ind}(v(A),t) ext^t_M X$.
(9) $ext^t_M GX = \{o \in OB \mid \text{for all } t' \ (t,t') \in R \text{ implies } o \in ext^{t'}_M X\}$.
(10) $ext^t_M FX = \{o \in OB \mid \text{there is a } t' \text{ such that } (t,t') \in R \text{ and } o \in ext^{t'}_M X\}$.
(11) $ext^t_M HX = \{o \in OB \mid \text{for all } t' \ (t',t) \in R \text{ implies } o \in ext^{t'}_M X\}$.
(12) $ext^t_M PX = \{o \in OB \mid \text{there is a } t' \text{ such that } (t',t) \in R \text{ and } o \in ext^{t'}_M X\}$.
Proof. immediate. //

Theorem 3.2.
(1) $ext^t_M [A]X = -ext^t_M <A> \neg X$.
(2) $ext^t_M GX = -ext^t_M F \neg X$.
(3) $ext^t_M HX = -ext^t_M P \neg X$.

We introduce the notions of *truth* and *validity* of formulas. A formula X is said to be true in model M ($\models_M X$) if for all $t \in TM$ $ext^t_M X = OB$. A formula X is said to be valid ($\models X$) if X is true in all models for logic TIL. Note that X is satisfiable iff $\neg X$ is not valid.

Theorem 3.3. The following conditions are equivalent:
(1) $\models_M X \rightarrow Y$.
(2) For $t \in TM$ $ext^t_M X \subseteq ext^t_M Y$.
Proof. $\models_M X \rightarrow Y$ iff $ext^t_M X \rightarrow Y = OB$ iff $-ext^t_M X \cup ext^t_M Y = OB$ iff $ext^t_M X \subseteq ext^t_M Y$. //

Theorem 3.4. The following conditions are equivalent:
(1) $\models_M <A>X \rightarrow [A]X$.
(2) $ext^t_M X$ is (v(A),t)-definable for all $t \in TM$.
Proof. By theorem 3.1(7) and (8), Theorem 3.3 and Theorem 2.2(1).

Theorem 3.5. For every model M, if $ind(v(B),t) \subseteq ind(v(A),t)$, then for every formula X, $\models_M [A]X \rightarrow [B]X$.
Proof. Since $ind(v(B),t) \subseteq ind(v(A),t)$, by Theorem 2.3(1), for any $S \subseteq OB$ $\underline{ind}(v(A),t)S \subseteq \underline{ind}(v(B),t)S$. Therefore, $-\underline{ind}(v(A),t) ext^t_M X \cup \underline{ind}(v(B),t) ext^t_M X = OB$. //

Using standard techniques of proposttional logic, modal logic [5] and tense logic [7], we can show the following.
Theorem 3.6.
(1) If $\models_M X$ and $\models_M X \rightarrow Y$, then $\models_M Y$.

(2) If $\models_M X$, then $\models_M [A]X$.
(3) If $\models_M X$, then $\models_M GX$.
(4) If $\models_M X$, then $\models_M HX$.

Let A, be arbitrary expression and let X and Y be arbitrary formulas. Then, we formulate some valid formulas of logic TIL.

Theorem 3.7. The following formulas are valid:

(1) All classical propositional tautologies.

(2) (a) $[A](X \to Y) \to ([A]X \to [A]Y)$.

(b) $[A]X \to X$.

(c) $X \to [A]\langle A\rangle X$.

(d) $[A]X \to [A][A]X$.

(3) (a) $G(X \to Y) \to (GX \to GY)$.

(b) $H(X \to Y) \to (HX \to HY)$.

(c) $X \to GPX$.

(d) $X \to HFX$.

(e) $GX \to GGX$.

(f) $HX \to HHX$.

(g) $FX \wedge FY \to (F(X \wedge Y) \vee F(X \wedge FY) \vee F(FX \wedge Y))$.

(h) $PX \wedge PY \to (P(X \wedge Y) \vee P(X \wedge PY) \vee P(PX \wedge Y))$.

Theorem 3.7(2a) and (3a), (3b) assure that logic TIL is normal. Theorem 3.7(2) and Theorem 3.6(1) and (2) have an obvious connection with the axiomatization of modal logic S5: for a fixed A, they form S5. Theorem 3.7(2b), (2c) and(2d) show reflexivity, symmetry and transitivity of the indiscernibility relations, respectively. Theorem 3.7(3) and Theroem 3.6(3) and (4) correspond to tense logic CL. Theorem 3.7(3c), (3d) say that past tense operations are inverse with future tense operations; (3e), (3f) express transitivity of time ordering and (3g), (3h) express forwards linearity and backwards linearity, respectively.

4. Decidability

In this section, we investigate the following important property of logic TIL: for any formula X, X is satisfiable if and only if X is satisfiable in a finite model for logic TIL (where both OB and TM have less than $2^{2|\Phi_X|} - 2^{|\Phi_X|} + 1$ elements, $|\Phi_X|$ is the cardinality of set Φ_X of all subformulas of X). The way of proving the finite model property is the method of filtration (cf. [3, 9]; in this paper, the model contains two classes of "worlds' and two classes of relations): given a model M and a formula X, we define another model M^* which is finite and in which for some object o and for some time t such that not $M \models_o^t X$, provided X is not true in M. Finally, we give a positive solution to the problem of deciding whether or not a formula in logic TIL is satisfiable (or valid).

Given a formula X and a model $M=(OB, TM, PROP, \{ind(P,t)\}_{P\in PROP, t\in TM}, R, v)$, we construct a model $M^*=(OB^*, TM^*, PROP^*, \{ind^*(P,t)\}_{P\in PROP^*, t\in TM^*}, R^*, v^*)$ in the following.

For any $o\in OB$ and any $t\in TM$, we define an equivalence relation $\cong$ in $OB\times TM$ by

$(o,t)\cong(o',t') \underset{}{\overset{def}{\Longleftrightarrow}}$ for any $Y\in\Phi_X$ $M\models^{t}_{o} Y$ iff $M\models^{t'}_{o'} Y$,

where Φ_X is the set of all subformulas of X (including X itself).

$[o,t] \overset{D}{=} \{(o',t')\in OB\times TM \mid (o,t)\cong(o',t')\}$. That is, we define $[o,t]$ as an equivalence class (containing the pair (o,t)) of the relation $\cong$. It is easily known that the number of all the classes $[o,t]$ can be limited within $2^{|\Phi_X|}$. It should be noticed that in general, if $(o',t')\in[o,t]$, then $[o',t']=[o,t]$; but if $(o,t')\notin[o,t]$ ($(o',t)\notin[o,t]$), then $[o,t]\neq[o,t']$ ($[o,t]\neq[o',t]$).

Moreover, $\Phi(o,t) \overset{D}{=} \{o'\in OB \mid$ for some t' such that $(o',t')\in[o,t]\}$, $\Psi(o,t) \overset{D}{=} \{t'\in TM \mid$ for some o' such that $(o',t')\in[o,t]\}$.

The components of M^* are as follows:

(1) $OB^* \overset{D}{=} \{o^*\subseteq OB \mid o^* = \bigcap_{o\in\Phi(o',t')} \Phi(o',t') - [(\bigcup_{\substack{o\notin\Phi(o'',t'') \\ \text{and} \\ \Phi(o'',t'')\cap(\bigcap_{o\in\Phi(o',t')} \Phi(o',t'))\neq\emptyset}} \Phi(o'',t'')) \cap (\bigcap_{o\in\Phi(o',t')} \Phi(o',t'))\,]\}$

(2) $TM^* \overset{D}{=} \{t^*\subseteq TM \mid t^* = \bigcap_{t\in\Psi(o',t')} \Psi(o',t') - [(\bigcup_{\substack{t\notin\Psi(o'',t'') \\ \text{and} \\ \Psi(o'',t'')\cap(\bigcap_{o\in\Psi(o',t')} \Psi(o',t'))\neq\emptyset}} \Psi(o'',t'')) \cap (\bigcap_{o\in\Psi(o',t')} \Psi(o',t'))\,]\}$.

(3) $PROP^*$ is the least set of $v(A)$ satisfying the following condition: for $A\in VARP$
$v(A)\in PROP^*$ if A occurs in a modal operation contained by a subformula of X;

(4) for $P\in PROP^*$ ($v(A)=P$), $t^*\in TM^*$, $(o^*,o'^*)\in ind^*(P,t^*)$ iff for any $[A]Y\in\Phi_X$ the following conditions are equivalent:

(i) for all $(x,t1)\in[o,t]\cap o^*\times t^*$ $M\models^{t1}_{x}[A]Y$,

(ii) for all $(x',t2)\in[o',t]\cap o'^*\times t^*$ $M\models^{t2}_{x'}[A]Y$;

(5) R^* in $TM^*\times TM^*$ is defined below:
if there is no tense operation in X, R^* is ordered under a transitive linear condition arbitrarily; otherwise, $(t^*,t'^*)\in R^*$ iff for any $o\in OB$ the following (i) and (ii) hold:

(i) for any $GY\in\Phi_X$ (a) implies (b):

(a) for all $(x,t1)\in o^*\times t^*\cap[o,t]$ $M\models^{t1}_{x} GY$

(b) for all $(y,t2)\in o^*\times t'^*\cap[o,t']$ $M\models^{t2}_{y} GY$ and $M\models^{t2}_{y} Y$;

(ii) for any $HY \in \Phi_X$ (c) implies (d):

(c) for all $(x,t2) \in o^* \times t'^* \cap [o,t']$ $M \models^{t2}_{x'} HY$

(d) for all $(y,t1) \in o^* \times t^* \cap [o,t]$ $M \models^{t1}_{y} HY$ and $M \models^{t1}_{y} Y$;

(6) v^*: VARO$\cup$VARP$\rightarrow \mathcal{P}$(OB*)$\cup$PROP* is a valuation function satisfying the following conditions:

for $Y \in$ VARO $v^*(Y) \subseteq OB^*$ such that for any $o^* \in v^*(Y)$ iff $o \in v(Y)$,

for $A \in$ VARP $v^*(A) \in PROP^*$ such that $v^*(A)=v(A)$ if $v(A) \in PROP^*$.

As defined above, all the elements of OB* are disjoint, and so are the elements of TM*. This consideration is meaningful in giving the definitions of ind*(P,t*) and R*. The definitions of OB* and TM* show that o* and t* contain, as least, o and t, respectively, and that OB* and TM* are nonempty since both OB and TM are nonempty. We evaluate the cardinalities of OB* and TM* just as we calculate the number of various intersections of 2^n circles. Therefore, the upper bound of OB* is $2^{2|\Phi_X|} - 2^{|\Phi_X|} + 1$; this result is suitable for TM*, too.

Lemma 4.1. For any $Y \in \Phi_X$ the following conditions are equivalent:

(1) $M \models^{t}_{o} Y$.

(2) For all $(o',t') \in [o,t] \cap o^* \times t^*$ $M \models^{t'}_{o'} Y$.

Proof. Suppose (1) holds. Then, there is an equivalence class [o,t] of $\cong$ such that for all $(o',t') \in [o,t]$ $M \models^{t}_{o} Y$ iff $M \models^{t'}_{o'} Y$. It follows that (2) holds. Suppose (1) does not hold. Then, for all $(o',t') \in [o,t]$, not $M \models^{t'}_{o'} Y$. Consequently, (2) does not hold. //

It should be stressed that $[o,t] \cap o^* \times t^* \neq o^* \times t^*$ and if (2) were: For all $(o',t') \in o^* \times t^*$ $M \models^{t'}_{o'} Y$, Lemma 4.1. would not hold.

Lemma 4.2.

(1) For $P \in PROP^*$ and $t^* \in TM^*$ ind*(P,t*) is an equivalence relation in OB*.

(2) For $P \in PROP^*$ if $(o,o') \in ind(P,t)$, then $(o^*,o'^*) \in ind^*(P,t)$.

(3) For $[A]Y \in \Phi_X$ if $(o^*,o'^*) \in ind^*(P,t^*)$, then, $M \models^{t}_{o} [A]Y$ implies $M \models^{t'}_{o''} Y$ for $(o'',t') \in [o',t] \cap o'^* \times t^*$.

Proof. (1) This proof is the matter of the definition of ind*(P,t*).

(2) Suppose $(o,o') \in ind(v(A),t)$. If $M \models^{t}_{o} [A]Y$, then $M \models^{t}_{o} [A][A]Y$ and by the assumption, $M \models^{t}_{o'} [A]Y$. Conversely, since $(o,o') \in ind(v(A),t)$ implies $(o',o) \in ind(v(A),t)$, similarly, we show that if $M \models^{t}_{o'} [A]Y$, then $M \models^{t}_{o} [A]Y$. By Lemma 4.1, $(o^*,o'^*) \in ind^*(v^*(A),t^*)$.

(3) Suppose $(o^*,o'^*) \in ind^*(v^*(A),t^*)$ and $M \models^{t}_{o} [A]Y$. Then, it can be derived from the definition of ind*(v*(A),t*) that $M \models^{t'}_{o''} [A]Y$ for $(o'',t') \in [o',t] \cap o'^* \times t^*$ and so, on

account of the reflexivity of ind(v(A)), we have $M \models^{t'}_{o''} Y$. //

Lemma 4.3.
(1) If $(t,t') \in R$, then $(t^*,t'^*) \in R^*$.
(2) R^* is a transitive linear ordering in TM^*.
(3) For any $GY \in \Phi_X$ if $(t^*,t'^*) \in R^*$, then for any $o \in OB$ $M \models^{t}_{o} GY$ implies $M \models^{t''}_{o'} Y$ for all $(o',t'') \in o^* \times t'^* \cap [o,t']$.
(4) For any $HY \in \Phi_X$ if $(t^*,t'^*) \in R^*$, then for any $o \in OB$ $M \models^{t'}_{o} FY$ implies $M \models^{t''}_{o'} Y$ for $(o',t'') \in o^* \times t^* \cap [o,t]$.

Proof. Let us make a comment on (1): if there is no tense operation in formula X, then, (1) does not hold. Indeed, we will apply (1) only in the case of existence of the tense operation (see the proof of Lemma 4.4).

(1) Assume that $(t,t') \in R$. By Theorem 3.7, $M \models^{t}_{o} GY$ implies $M \models^{t}_{o} GGY$. By the assumption, $M \models^{t}_{o} GY$ implies $M \models^{t'}_{o} Y$, and $M \models^{t}_{o} GGY$ implies $M \models^{t'}_{o} GY$. Similarly, we show that for any $HY \in \Phi_X$ if $M \models^{t'}_{o} HY$, then $M \models^{t}_{o} HY$ and $M \models^{t}_{o} Y$. Thus, by Lemma 4.1, we have $(t^*,t'^*) \in R^*$.

(2) The transitivity of R^* follows from the definition of R^*. Supposes there is a pair (t^*,t'^*) in $TM^* \times TM^*$ such that $(t^*,t'^*) \notin R^*$ and $(t'^*,t^*) \notin R^*$ and $t^* \neq t'^*$. Then by (1), $(t,t') \notin R$ and $(t',t) \notin R$ and $t \neq t'$, a contradiction. Therefore, R^* is linear.

(3) Assume that $(t^*,t'^*) \in R^*$. Suppose $M \models^{t}_{o} GY$. Then, by the definition of R^*, we have $M \models^{t''}_{o'} Y$ for all $(o',t'') \in o^* \times t'^* \cap [o,t']$.

(4) This is similar to the proof of (3). //

By a *finite model* for TIL, we mean a model $M=(OB, TM, PROP, \{ind(P,t)\}_{P \in PROP, t \in TM}, R, v)$ for TIL, where both OB and TM are finite. In this sense, the previous discussion makes it clear that the model M^* is a finite model for TIL.

Lemma 4.4. For any $Y \in \Phi_X$, any $o \in OB$ and any $t \in TM$ $M \models^{t}_{o} Y$ iff $M^* \models^{t^*}_{o^*} Y$.
Proof. This proof is by induction on the structure of Y. //

We call M^* *filtration* of M when Lemma 4.4 holds.

Theorem 4.5. X is satisfiable iff X is satisfiable in a finite model where $|OB| \leq 2^{2|\Phi_X|} - 2^{|\Phi_X|} + 1$ and $|TM| \leq 2^{2|\Phi_X|} - 2^{|\Phi_X|} + 1$.
Proof. It is sufficient to show the "only if" part only: if X is not satisfiable in any such finite model, then, X is not satisfiable in any model. Suppose the claim does not hold, namely, there is a model M (without loss of generality, M can be assumed to be not such a finite model) such that X is satisfiable in M. Then, by Lemma 4.4, X is satisfiable in M^*, a contradiction. //

The finite model property has an important bearing on decidability. This means that there are finitely many finite models where $|OB| \leq 2^{2|\Phi x|} - 2^{|\Phi x|} + 1$ and $|TM| \leq 2^{2|\Phi x|} - 2^{|\Phi x|} + 1$, and that given a formula X and such a model, it is possible to decide effectievly whether or not X is satisfiable (or valid) in the model.

As desired, we reach a conclusion on the decidability of logic TIL.

Corollary 4.6. The satisfiability problem(or the validity problem) of formulas in logic TIL is solvable.

5. Future topic

We have proposed a logic TIL of temporal information, and shown the decidability of TIL. In fact, in addition to the indiscernibility, there are many relations depending on properties of objects that can be used to induce patterns in a set of objects. A question is how to completely axiomatize these logics like TIL with respect to "2 dimensional" worlds.

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